

Carr Hill Community Primary School  
Developing calculation across Key Stages 1 and 2

### Introduction

This document recommends an approach for developing progression in the conceptual and procedural aspects of calculation across Key Stages 1 and 2. It takes into account the mathematics programmes of study and non-statutory guidance for the National Curriculum 2014.

The document covers:

- Calculations that can be done wholly or partially by mental methods, based on fluency with number facts and understanding of place value and number operations and sometimes using horizontally presented number sentences or empty number lines to show steps in thinking.
- The use of expanded or informal written methods to support understanding of compact, formal written methods and
- Developing fluency in the use of formal written methods by the end of key stage 2.
- Appropriate use of a calculator.

Children should work towards being able to use, by the end of Key Stage 2:

- A range of strategies for mental calculations *appropriate to the numbers involved*.
- One formal written method (for each number operation) for calculations that cannot be done mentally.
- A calculator for calculations where this is the most appropriate choice.

Progression in mental calculation skills can be supported by:

- The ability to quickly recall a range of number facts and an understanding of how to use them to derive other related facts.
- Understanding how numbers and calculations can be represented by materials and images such as arrays, ten frames, Numicon shapes.
- An understanding of the number system (order and relative position of numbers, place value, etc), the four number operations and the laws of arithmetic associated with them
- Understanding of how symbols are used to record calculations especially the equals sign. Care should be taken that the equals sign is used correctly
  - Eg  $42 + 35 =$  might be calculated by partitioning the second number to add the tens followed by the units. This could be recorded as:  
 $42 + 30 = 72$   
 $72 + 5 = 77$   
*But not as  $42 + 30 = 72 + 5 = 77$*   
as this involves an incorrect use of the first equals sign.
- An understanding of how calculations can be represented on empty number lines. They will need to work with numbered tracks and lines first before they are confident to rely on empty lines alone. To make good use of empty lines children need to be able to:
  - Move forward and back confidently on the number line.
  - Make jumps of different sizes.
  - Recognize landmark numbers such as multiples of 10
  - **Know** and use number complements to 10 and how these relate to multiples of 10.
  - Partition and recombine numbers in appropriate ways eg  $7 + 5$  as  $7 + 3 + 2$ , or  $28 + 9$  as  $28 + 10 - 1$ .

Teachers should **demonstrate** the use of number sentences and number lines to **model** steps in calculations. Children should be encouraged to record the steps in their mental calculations some of the time. Recording is useful when explaining methods to others and to show which strategy has been used. It is not necessary to always record, especially for those children who have efficient mental methods. Teachers should use their judgement about when to require recording.

Progression to fluency with a formal written method for each number operation can be made by

- The appropriate use of informal or expanded written methods that build on mental methods and which continue to highlight understanding of the number system and number operations.
- Linking of these expanded methods to the formal written method when it is first introduced to highlight steps that may be concealed, and hence not understood, in the procedural execution of the formal written method.
- Appropriate levels of practice of formal written methods to develop fluency.

Children should continue to develop their mental calculation skills with larger numbers once written methods are introduced and should be given opportunities to identify which calculations might be done mentally, with reference to the **nature** rather than **magnitude** of the numbers involved. They should use mental calculation skills to estimate the likely magnitude of the answer when performing a calculation using a formal written method and hence identify answers that are unreasonable and indicate errors in execution of the method.

Teachers need to judge when children are ready to move from mental to written calculations. The following lists offer some guidance.

### **Addition and subtraction**

#### **Can pupils:**

- recall addition and subtraction facts to 20?
- understand place value and partition numbers?
- add three single digit numbers mentally?
- add or subtract any pair of two digit numbers mentally?
- explain their mental strategies orally and record them using horizontal number sentences or an empty number line?

### **Multiplication and division**

#### **Can pupils:**

- quickly recall multiplication and division facts for 2, 3, 4, 5 and 10 times tables?
- understand what happens when a number is multiplied by 0 or 1?
- understand 0 as a place holder?
- multiply two- and three-digit numbers mentally by 10 and 100?
- demonstrate understanding of the commutative, distributive and associative laws (though not necessarily know the names)?
- double and halve two-digit numbers mentally?
- explain mental strategies orally and with recording?

This document considers addition and subtraction together followed by multiplication and division. **Links between number operations should be emphasised regularly.** A year-by-year approach has been taken in line with the format of the National Curriculum 2014 programmes of study, but teachers should have regard to other year group expectations when planning for different abilities.

## Addition and subtraction

The first table below gives an overview of the calculation expectations for each year group. Statements highlighted in **bold** can be matched to the National Curriculum 2014 programmes of study or non-statutory guidance. Other items support children's understanding of calculation methods.

The second table in this section sets of how children's recording of calculations might look depending on the mental strategy or written method being used.

Year	Addition and subtraction
1	<p>Children in Year 1 should:</p> <ul style="list-style-type: none"> <li>• <b>Use concrete objects and pictorial representations, including number lines, to support their solution of addition and subtraction problems.</b></li> <li>• <b>Represent and use number bonds and related subtraction facts within 20, memorizing and reasoning with these bonds.</b></li> <li>• <b>Add and subtract one-digit and two-digit numbers to 20, including zero (and realize the effect of adding or subtracting zero to establish the relationship between these operations)</b></li> <li>• <b>Read, write and interpret mathematical statements involving addition (+), subtraction (-) and (=) signs in a range of formats e.g. <math>\Delta + 5 = 12</math> or <math>7 = \diamond - 9</math></b></li> </ul>
2	<p>Children in Year 2 should:</p> <ul style="list-style-type: none"> <li>• <b>Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100 e.g. use <math>3 + 7 = 10</math> to derive <math>30 + 70 = 100</math></b></li> <li>• <b>Use concrete objects and pictorial representations to support their solution of addition and subtraction problems and to add and subtract mentally including TU+/-U, TU+/- T, TU +/- TU, U + U + U.</b></li> <li>• <b>Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot. Use the associative law of addition to show for example that <math>5 + 2 + 1 = 1 + 5 + 2 = 1 + 2 + 5</math></b></li> <li>• <b>Recognize and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems</b></li> <li>• Record mental additions and subtractions using horizontal number sentences and/or empty number lines to show and explain the steps in their calculations.</li> <li>• <b>Recording in columns supports place value and prepares for formal methods.</b></li> </ul>
3	<p>Children in Year 3 should:</p> <ul style="list-style-type: none"> <li>• <b>Add and subtract numbers mentally including HTU +/- U, HTU+/- T, HTU+/- H</b></li> <li>• Use horizontal number sentences and empty number lines sometimes to support explanation of their mental calculation methods.</li> <li>• <b>Solve varied addition and subtraction problems including missing number problems using number facts and place value.</b></li> <li>• Develop their understanding of written methods; working from expanded to <b>using (compact) formal written methods of columnar addition and subtraction with numbers of up to three digits.</b> Particular attention should be paid to the language used when modeling these methods. The value of digits should be retained according to place value and use of practical materials /representations may aid understanding</li> <li>• <b>Estimate the answer to a calculation and check using inverse operations.</b></li> </ul>
4	<p>Children in Year 4 should:</p> <ul style="list-style-type: none"> <li>• <b>Continue to add and subtract numbers with up to four digits mentally</b> where the nature of the numbers makes this appropriate. They may use horizontal number sentences or empty number lines to support an explanation of the steps in their calculation. They should be given opportunities to identify calculations which are appropriate for a mental method and explain why.</li> <li>• <b>Add and subtract numbers with up to four digits using the formal written methods of columnar addition and subtraction where appropriate.</b> Their understanding of the procedures involved may be supported by the use of expanded written methods and practical materials if required.</li> <li>• <b>Estimate and use inverse operations to check answers to a calculation.</b></li> </ul>

5	<p>Children in Year 5 should:</p> <ul style="list-style-type: none"> <li>• <b>Add and subtract numbers mentally with increasingly large numbers e.g. <math>12,462 - 2300 = 10,162</math>.</b> Use horizontal number sentences and empty number lines sometimes to support explanation of their methods. They should be given opportunities to identify calculations which are appropriate for a mental method and explain why.</li> <li>• <b>Add and subtract whole numbers with more than four digits, including using formal written methods (columnar addition and subtraction).</b> Particular attention should be paid to the language used when modelling these methods. The value of digits should be retained according to their place value. Understanding of the procedures involved may be supported by the use of expanded written methods and practical materials if required.</li> <li>• <b>Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy</b></li> <li>• <b>Solve addition and subtraction multi-step problems in context, deciding which operations to use and why.</b></li> <li>• Learn how to record the method they used when working with a calculator.</li> </ul>
6	<p>Children in Year 6 should:</p> <ul style="list-style-type: none"> <li>• <b>Perform mental calculations including with mixed operations and large numbers (and decimals).</b> Use horizontal number sentences and empty number lines sometimes to support explanation of their methods. They should be given opportunities to identify the most appropriate tool for calculations ie mental method, mental with recording, formal written method or calculator and explain why.</li> <li>• <b>Practise addition and subtraction for (appropriate) larger numbers and decimals using the formal written methods of columnar addition and subtraction.</b> Those who are not able to use the compact formal method may use an expanded method and work towards an understanding of the formal written method. Particular attention should be paid to the language used when modelling these methods. The value of digits should be retained according to their place value. Materials / representations may support understanding.</li> <li>• <b>Use estimation to check answers to calculations and determine, in the context of the problem, an appropriate degree of accuracy.</b></li> <li>• <b>Round answers to a specified degree of accuracy.</b></li> <li>• <b>Use knowledge of the order of operations, and use of brackets, to carry out calculations involving the four operations.</b></li> <li>• <b>Solve addition and subtraction multi-step problems in contexts, deciding which operations to use and why.</b></li> <li>• Learn how to record the method they used when working with a calculator.</li> </ul>

## Multiplication and Division

Children should develop understanding of multiplication as:

- repeated addition
- describing an array
- scaling

And an understanding of division as:

- grouping
- sharing

Children can develop this understanding and perform calculations through recording in a variety of ways:

- drawing pictures and making marks
- drawing and partitioning arrays
- drawing jumps on number lines
- writing number sentences and using informal and formal written methods

The first table below gives an overview of the calculation expectations for each year group. Statements highlighted in **bold** can be matched to the National Curriculum 2014 programmes of study or non-statutory guidance. Other items support children's understanding of calculation methods.

The second table in this section sets out how children's recording of calculations might look depending on the mental strategy or written method being used.

Year	Multiplication and division
1	<p>Children in Year 1 should:</p> <ul style="list-style-type: none"> <li>• Solve one step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</li> <li>• Through grouping and sharing small quantities, pupils begin to understand: multiplication and division; doubling numbers and quantities; and finding simple fractions of numbers and quantities</li> <li>• Pupils make connections between arrays, number patterns and counting in twos, fives and tens.</li> </ul>
2	<p>Children in Year 2 should:</p> <ul style="list-style-type: none"> <li>• Use materials and contexts in which multiplication and division relate to grouping and sharing discrete and continuous quantities, to arrays and to repeated addition. Begin to relate ideas to fractions and measures</li> <li>• Recall and use the multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.</li> <li>• Make connections between these tables and connect the 10 multiplication table to place value and the five table to divisions on a clock face</li> <li>• Use number sentences to show multiplication as repeated addition.</li> <li>• Record multiplications and divisions as jumps on number lines.</li> <li>• Calculate mathematical statements for multiplication and division within the multiplication tables and use <math>\times</math>, <math>\div</math> and <math>=</math> signs.</li> <li>• Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.</li> <li>• Use commutativity and inverse relationships to develop multiplicative reasoning e.g. <math>4 \times 5 = 20</math> and <math>20 \div 5 = 4</math></li> <li>• Solve problems, including problems in contexts, involving multiplication and division, using materials, arrays, repeated addition, mental methods and multiplication and division facts.</li> </ul>
3	<p>Children in Year 3 should:</p> <ul style="list-style-type: none"> <li>• Draw pictures and arrays to represent multiplications and divisions if necessary to support understanding, including for situations involving remainders.</li> <li>• Use number sentences and / or number lines to explain multiplication / division as repeated addition / subtraction</li> <li>• Partition arrays to find related number facts for single digit tables facts eg <math>8 \times 4 = (4 + 4) \times 4</math> or <math>8 \times 4 = (5 + 3) \times 4</math>.</li> <li>• Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables. Connect 2, 4 and 8 tables through doubling.</li> <li>• Develop efficient mental methods e.g. using commutativity and associativity and using known facts to derive other related facts.</li> <li>• Use partitioning and use of the distributive law to introduce multiplication and division of a two digit by one digit number. Support this work with images and materials such as arrays and place value counters.</li> <li>• Write and calculate mathematical statements for multiplication and division using the multiplication statements that they know, including for two-digit numbers times one digit numbers, using mental and progressing to formal written methods of short multiplication and division.</li> <li>• Use tables facts to solve problems including missing number, integer scaling and correspondence problems in which <math>n</math> objects are connected to <math>m</math> objects</li> </ul>

4	<p>Children in Year 4 should:</p> <ul style="list-style-type: none"> <li>• Recall multiplication and division facts for multiplication tables up to 12 x 12</li> <li>• Explore division situations that give rise to remainders</li> <li>• Use place value, known and derived facts to multiply and divide mentally (e.g. <math>600 \div 3 = 200</math> can be derived from <math>2 \times 3 = 6</math>), including multiplying by 0 and 1; dividing by 1; multiplying together three numbers.</li> <li>• Use knowledge of number facts and laws of arithmetic: commutative, associative and distributive to solve mental and written calculations.</li> <li>• Recognize and use factor pairs</li> <li>• Use arrays and models such as the grid method or place value counters to develop understanding of the formal methods of short multiplication and division</li> <li>• Multiply two digit and three digit numbers by a one digit number using formal written layout of short multiplication</li> <li>• Use the formal written method of short division with exact answers.</li> <li>• Solve one and two step problems in contexts involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and correspondence problems such as when n objects are connected to m objects.</li> </ul>
5	<p>Children in Year 5 should:</p> <ul style="list-style-type: none"> <li>• Apply all multiplication tables and related division facts frequently, commit them to memory and use them confidently to multiply and divide numbers mentally to make larger calculations.</li> <li>• Develop understanding and use of factors, multiples, factor pairs, common factors and multiples, primes, prime factors, non primes (composite numbers), squares and cubes (including notation for these). Establish if a number up to 100 is prime and recall primes to 19.</li> <li>• Multiply numbers up to 4 digits by a one or two digit number using a formal written method, including long multiplication for two-digit numbers</li> <li>• Divide numbers up to 4 digits by a one digit number using the formal written method of short division and interpret remainders appropriately for the context, including as fractions, decimals or by rounding.</li> <li>• Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000</li> <li>• Use an expanded / informal method if they are not ready for the formal methods and be supported towards an understanding of the compact method using e.g. the grid method or place value counters</li> <li>• Use multiplication and division facts to solve problems involving scaling by simple fractions and problems involving simple rates</li> <li>• Learn how to record the <i>method</i> they used when working with a calculator.</li> </ul>
6	<p>Children in Year 6 should:</p> <ul style="list-style-type: none"> <li>• Perform mental calculation, including with mixed operations and large numbers. Identify common factors, common multiples and prime numbers.</li> <li>• Multiply numbers up to four digits by a two digit whole number using the formal written method of long multiplication</li> <li>• Divide numbers up to four digits by a two digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context.</li> <li>• Divide numbers up to four digits by a two digit whole number using the formal written method of short division where appropriate, and interpret remainders according to the context.</li> <li>• Use knowledge of the order of operations to carry out calculations involving the four operations.</li> <li>• Solve problems in context using all four number operations and determine, in the context of the problem, an appropriate degree of accuracy</li> </ul>

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|--|---|
|  | <ul style="list-style-type: none"><li>• They should be given opportunities to identify the most appropriate tool for calculations ie mental method, mental with recording, standard method or calculator and explain why.</li><li>• Use compact formal methods if they can do so efficiently and with understanding. Those who are not able to use a standard method should use an expanded method and work towards an understanding of a compact method.</li><li>• Learn how to record the <i>method</i> they used when working with a calculator.</li></ul> |
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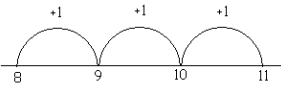
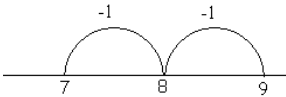
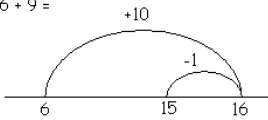
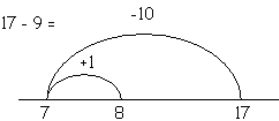


## Calculation Methods for Year 1

### YEAR 1

**NOTE:** Pupils should memorise and reason with number bonds to 10 and 20. Use of structured materials such as ten frames may support this and reduce dependence on count by ones strategies.

Pupils should become familiar with the terms: *put together, add, altogether, total, take away, distance between, difference between, more than and less than* so they develop the concept of addition and subtraction and can use these operations flexibly.

Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
Counting on/back	$8 + 3 = 11$		$9 - 2 = 7$	
Reordering: Count on from larger number	$3 + 8 =$ ✍️ rewrite as $8 + 3 = 11$	Reorder before using number line as above		
Find pairs that total 10	$3 + 4 + 7 =$ $3 + 7 + 4 =$ $10 + 4 = 14$			
Add or subtract 9	$6 + 9 =$ ✍️ $6 + 10 - 1 =$ $16 - 1 = 15$	$6 + 9 =$ 	$17 - 9 =$ ✍️ $17 - 10 + 1 =$ $=$ $7 + 1 = 8$	$17 - 9 =$ 

Year 1

Multiplication

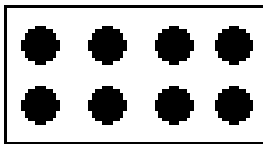
Pictures/marks

There are 3 sweets in one bag.  
How many sweets are there in 5 bags?

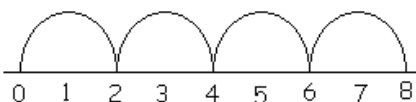
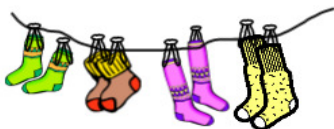


Arrays

2 groups of 4 or 4 groups of two



Jumps forward on a number line



Division

Pictures/marks

12 children get into teams of 4 to play a game.  
How many teams are there?

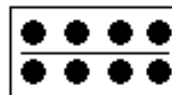


Arrays

Put into groups of two



Share between two



Jumps backwards on a number line

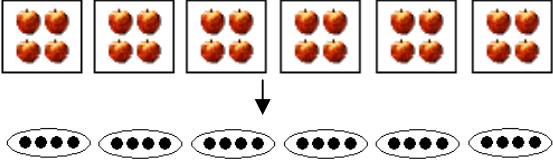
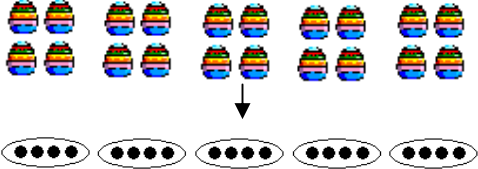

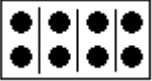
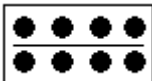
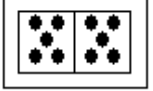
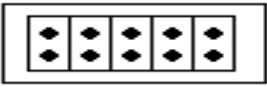
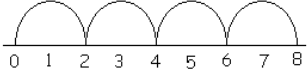
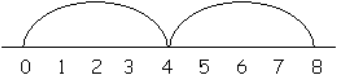
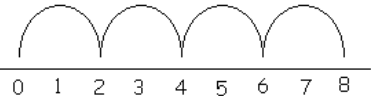
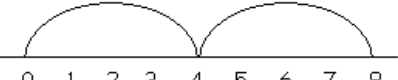
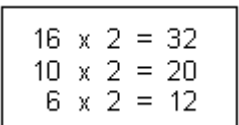
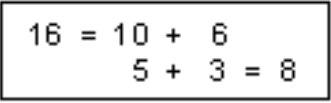
## Calculation Methods for Year 2

### YEAR 2

**Establish the use of efficient, non counting based, strategies using bonds to 20, place value etc.**

Use of representations and materials such as ten frames and base ten materials may support understanding.

Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
Counting on/back in ones and tens. <b>Move children to using known bonds to reduce reliance on this.</b>	$34 + 3 =$ $23 + 20 =$	 	$27 - 4 =$ $45 - 20 =$	 
Count up to find a small difference			$82 - 79 =$	
Reordering	$5 + 7 + 5 =$ $5 + 5 + 7 =$			
Use near doubles	$6 + 7 =$ $6 + 6 + 1 =$			
Partitioning using multiples of 10. <i>Partition both numbers or just the second.</i>	$25 + 14 =$ $20 + 10 = 30$ $5 + 4 = 9$ or $25 + 10 = 35$ $35 + 4 = 39$		$46 - 23 =$ $40 - 20 = 20$ $6 - 3 = 3$ or $46 - 20 = 26$ $26 - 3 = 23$ NB In cases such as $43 - 26 =$ <del><math>30 - 20 =</math></del> <del><math>13 - 6 =</math></del>	
Compensating to add/subtract numbers close to a multiple of 10	$24 + 19 =$ $24 + 20 - 1 =$  $58 + 21 =$ $58 + 20 + 1 =$	 	$70 - 11 =$ $70 - 10 - 1 =$  $53 - 19 =$ $53 - 20 + 1 =$	 



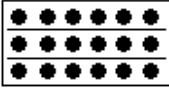
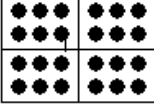
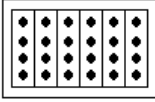
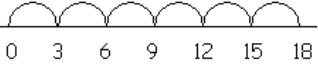
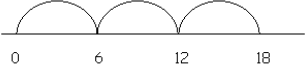
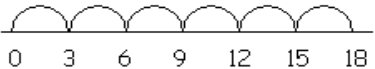
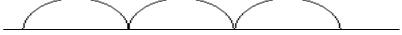
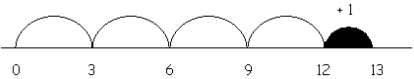
Multiplication	Division
<p><b>Pupils should work towards fluent recall of 2, 5 and 10 multiplication tables and derivation of related division facts and use these to calculate and solve problems. They may be supported by the type of activities shown below and regular practice of tables facts. They could explore other tables in these ways.</b></p>	
<p><b>Pictures/marks</b></p> <p>There are 4 apples in one box. How many apples in 6 boxes?</p> 	<p><b>Pictures/marks</b></p> <p>4 eggs fit in a box. How many boxes would you need to pack 20 eggs?</p> 
<p><b>Other Jottings</b></p> <p>Arrays  <math>2 \times 4</math> or <math>4 \times 2</math></p>  <p>Repeated addition  <math>2 \times 4 = 2 + 2 + 2 + 2</math></p> <p>or  <math>2 = 4 + 4</math>      <math>4 \times</math></p>	<p><b>Other Jottings</b></p> <p>Arrays <math>8 \div 2</math></p>  or  <p>Sharing <math>10 \div 2</math></p>  <p>Grouping <math>10 \div 2</math></p> 
<p><b>Number Lines</b> (numbered then empty)</p> <p><math>2 \times 4</math></p>  <p><math>4 \times 2</math></p>  <p>Recording by</p> <ul style="list-style-type: none"> <li>- drawing jumps on prepared line</li> <li>- constructing own lines</li> </ul>	<p><b>Number Lines</b> (numbered then empty)</p> <p><math>8 \div 2</math></p>  <p><math>8 \div 4</math></p>  <p>Recording by</p> <ul style="list-style-type: none"> <li>- drawing jumps on prepared lines</li> <li>- constructing own lines</li> </ul>
<p><b>Signs and symbols</b></p> <p><math>\text{pencil} \times 2 = \text{pencil}</math>      <math>\text{pencil} = 2 \times 6</math>  <math>6 \times \text{pencil} = 12</math>      <math>12 = \text{pencil} \times 6</math>  <math>\text{pencil} \times 2 = 12</math>      <math>12 = \text{pencil} \times \text{pencil}</math></p>	<p><b>Signs and Symbols</b></p> <p><math>12 \div 2 = \text{pencil}</math>      <math>\text{pencil} = 12 \div 2</math>  <math>12 \div \text{pencil} = 6</math>      <math>6 = \text{pencil} \div 2</math>  <math>\text{pencil} \div 2 = 6</math>      <math>6 = 12 \div \text{pencil}</math>  <math>\text{pencil} \div \text{pencil} = 6</math>      <math>6 = \text{pencil} \div \text{pencil}</math></p> <p>Extend to <math>15 - 10 = 10 \div \text{pencil}</math></p>
<p><b>Doubling by partitioning</b></p> 	<p><b>Halving by partitioning</b></p> 

## Calculation Methods for Year 3

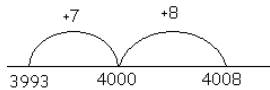
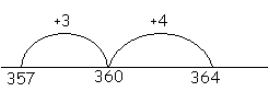
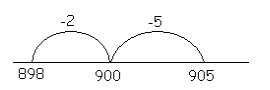
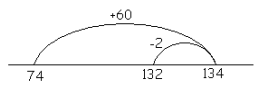
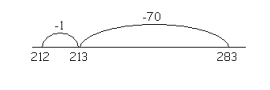
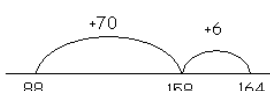
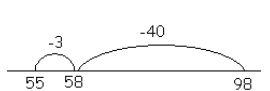
YEAR 3				
Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
<p>Children should calculate mentally with up to three digit numbers when nature of numbers makes this appropriate. They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but may be able to work without jottings e.g. <math>433 + 200</math>, <math>385 - 40</math>, <math>501 - 4</math>. They may use the following strategies and recording.</p>				
Add/subtract ones, tens and hundreds using number bonds and place value to find most efficient steps.	$90 + 40 =$		$110 - 30 =$	
Count up to find a difference			$504 - 498 =$	
Compensating to add/subtract numbers close to a multiple of 10	$543 + 29 =$ $543 + 30 - 1 =$		$273 - 29 =$ $273 - 30 + 1 =$	
Partitioning using multiples of 10 <i>Partition both numbers or just the second</i>	$86 + 57 =$ $80 + 50 = 130$ $6 + 7 = 13$ or $86 + 50 = 136$ $136 + 7 = 143$		$96 - 24 =$ $90 - 20 = 70$ $6 - 4 = 2$ or $96 - 20 = 76$ $76 - 4 = 72$	

### Introducing the formal written method of columnar addition and subtraction

	Addition	Subtraction
Partitioning both numbers using multiples of 10 and using the expanded method my help children move to the formal written method with understanding.	$\begin{array}{r} 67 \\ + 24 \\ \hline 11 \text{ (7+4)} \\ \underline{80} \text{ (60+20)} \\ 91 \end{array}$	$\begin{array}{r} 87 \\ - 53 \\ \hline 7 \text{ (60)} \\ 20 \text{ (80)} \\ \underline{7} \text{ (87)} \\ 34 \\ 1 \end{array}$
Introduce the <b>formal method</b> with or without regrouping as appropriate for pupils	$\begin{array}{r} 234 \\ +145 \\ \hline 379 \end{array}$ $\begin{array}{r} 234 \\ +178 \\ \hline 412 \\ 11 \end{array}$	

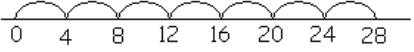
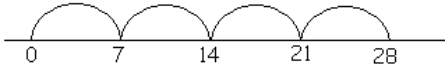
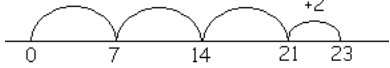
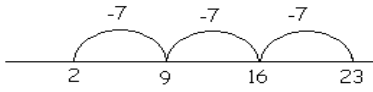
Multiplication	Division																
<p>The focus should be on <b>recall and use of the multiplication and division facts for the 3, 4 and 8 multiplication tables.</b> They connect the 2, 4 and 8 multiplication tables through doubling. They may use activities and recording shown below to develop understanding and promote fluency.</p>																	
<p><b>Arrays</b></p> <p>Arrays  <math>3 \times 6</math> or <math>6 \times 3</math></p>  <p>Repeated addition  <math>3 \times 6 = 3 + 3 + 3 + 3 + 3 + 3</math></p> <p>or  <math>6 \times 3 = 6 + 6 + 6</math></p>	<p><b>Arrays</b></p> <p>Arrays <math>18 \div 3</math></p>  <p>or</p>  <p>Sharing <math>24 \div 4</math></p>  <p>Grouping <math>24 \div 4</math></p> 																
<p><b>Number Lines</b></p> <p><math>3 \times 6</math></p>  <p><math>6 \times 3</math></p> 	<p><b>Number Lines</b></p> <p><math>18 \div 3</math></p>  <p><math>18 \div 6</math></p>  <p>Remainders  <math>13 \div 3</math></p> 																
<p><b>Write and calculate mathematical statements for multiplication and division.</b></p>																	
<p><math>6 \times 8 = \text{pencil}</math>      <math>\text{pencil} = 4 \times 6</math>  <math>6 \times \text{pencil} = 30</math>      <math>30 = \text{pencil} \times 6</math>  <math>\text{pencil} \times 4 = 28</math>      <math>16 = 2 \times \text{pencil}</math>  <math>\text{pencil} \times \text{pencil} = 24</math>      <math>12 = \text{pencil} \times \text{pencil}</math></p>	<p><math>30 \div 5 = \text{pencil}</math>      <math>\text{pencil} = 24 \div 6</math>  <math>12 \div \text{pencil} = 6</math>      <math>6 = \text{pencil} \div 5</math>  <math>\text{pencil} \div 4 = 7</math>      <math>8 = 40 \div \text{pencil}</math>  <math>\text{pencil} \div \text{pencil} = 3</math>      <math>4 = \text{pencil} \div \text{pencil}</math></p>																
<p><b>Calculate two digit numbers times one digit numbers using mental and progressing to formal written methods.</b>  This could be done by developing the understanding of arrays to introduce the grid method which can then be related to the formal method.</p> <p>Grid method (use vertical method to support movement into expanded method)</p> <p><math>43 \times 3 =</math></p> <table border="1" data-bbox="272 1496 424 1603"> <tr> <td>x</td> <td>3</td> </tr> <tr> <td>40</td> <td>120</td> </tr> <tr> <td>3</td> <td>9</td> </tr> </table> <p>= 129</p>	x	3	40	120	3	9											
x	3																
40	120																
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<p><math>43 \times 3</math></p> <table style="margin-left: 100px;"> <tr> <td></td> <td>43</td> </tr> <tr> <td></td> <td><u>x 3</u></td> </tr> <tr> <td>3 x 3 =</td> <td>9</td> </tr> <tr> <td>40 x 3 =</td> <td><u>120</u></td> </tr> <tr> <td></td> <td><u>129</u></td> </tr> </table>		43		<u>x 3</u>	3 x 3 =	9	40 x 3 =	<u>120</u>		<u>129</u>	<p>Leading to</p> <table style="margin-left: 100px;"> <tr> <td></td> <td>43</td> </tr> <tr> <td></td> <td><u>x 3</u></td> </tr> <tr> <td></td> <td><u>129</u></td> </tr> </table>		43		<u>x 3</u>		<u>129</u>
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## Calculation Methods for Year 4

YEAR 4				
Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
Children should calculate mentally with up to four digit numbers when nature of numbers makes this appropriate. They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but may be able to work without jottings e.g. $1433 + 200$ , $985 - 420$ , $1510 - 40$ . They may use the following strategies.				
Count up to find a small difference			$403 - 386 =$ $4008 - 3993 =$	
Bridge through multiples of 10	$357 + 7 =$ $357 + 3 + 4 =$		$905 - 7 =$ $905 - 5 - 2 =$	
Compensating to add/subtract numbers close to a multiple of 10	$74 + 58 =$ $74 + 60 - 2 =$		$283 - 71 =$ $283 - 70 - 1 =$	
Partitioning using multiples of 10 <i>Partition both numbers or just the second.</i>	$88 + 76 =$ $80 + 70 = 150$ $8 + 6 = 14$ or $88 + 70 = 158$ $158 + 6 = 164$		$98 - 43 =$ $90 - 40 = 50$ $8 - 3 = 5$ or $98 - 40 = 58$ $58 - 3 = 55$	

### Developing the formal written method of columnar addition and subtraction

	Addition	Subtraction
Most children should add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction.		
For children who are not ready for these formal methods use: <ul style="list-style-type: none"> <li>the informal expanded method for addition, adding the least significant digits first and</li> <li>the expanded decomposition method for subtraction</li> </ul> Move from these when ready to the <b>formal written method</b> supporting children to understand regroupings involved. Place value materials and representations may support children to understand the written procedure if required	$\begin{array}{r} 358 \\ + 73 \\ \hline 11 \\ 120 \\ \hline 300 \\ \hline 431 \end{array}$	$\begin{array}{r} 358 \\ + 73 \\ \hline 431 \\ \hline 11 \end{array}$
		$\begin{array}{r} 754 \\ - 36 \\ \hline 4 \text{ (40)} \\ 60 \text{ (100)} \\ \hline 654 \text{ (754)} \\ \hline 714 \\ \hline 1 \end{array}$

Multiplication	Division									
<p>Pupils should recall multiplication and division facts for multiplication tables up to 12 x 12. They may use activities and recording shown below to develop understanding and promote fluency.</p>										
<p><b>Use number lines</b></p> <p><math>4 \times 7 =</math> </p> <p><math>7 \times 4</math>  =</p>	<p><b>Number lines (including remainders)</b></p> <p><math>23 \div 7</math></p> <p>or </p> <p><math>23 \div 7</math></p> 									
<p>Record multiplication and division facts</p>										
<p><math>3 \times 7 = \text{pencil}</math>      <math>\text{pencil} = 7 \times 3</math>  <math>3 \times \text{pencil} = 21</math>      <math>21 = \text{pencil} \times 3</math>  <math>\text{pencil} \times 7 = 21</math>      <math>21 = 7 \times \text{pencil}</math>  <math>\text{pencil} \times \text{pencil} = 21</math>      <math>21 = \text{pencil} \times \text{pencil}</math></p>	<p><math>21 \div 7 = \text{pencil}</math>      <math>\text{pencil} = 21 \div 7</math>  <math>21 \div \text{pencil} = 3</math>      <math>3 = \text{pencil} \div 7</math>  <math>\text{pencil} \div 7 = 3</math>      <math>3 = 21 \div \text{pencil}</math>  <math>\text{pencil} \div \text{pencil} = 3</math>      <math>3 = \text{pencil} \div \text{pencil}</math></p>									
<p><b>Use place value, known and derived facts to multiply and divide mentally.</b>  <math>30 \times 6 = 3 \times 6 \times 10 = 18 \times 10 = 180</math></p>										
<p><b>Multiply two and three digit numbers by a one digit number using the formal written layout</b></p> <p>They may use informal or expanded methods to support understanding of the formal written method.</p> <p>Grid method (use vertical method to support movement into expanded method)</p> <p><math>23 \times 7 =</math></p> <table border="1" data-bbox="268 1079 422 1187"> <tr><td>x</td><td>7</td></tr> <tr><td>20</td><td>140</td></tr> <tr><td>3</td><td>21</td></tr> </table> <p>=161</p>	x	7	20	140	3	21				
x	7									
20	140									
3	21									
<p>Expanded method (x by units first to lead into formal method)</p> <p><math>23 \times 7 =</math></p> <table style="margin-left: 40px;"> <tr><td>TU</td></tr> <tr><td>23</td></tr> <tr><td><u>x 7</u></td></tr> <tr><td>21 (3x7)</td></tr> <tr><td>140 20x7</td></tr> <tr><td><u>161</u></td></tr> </table>	TU	23	<u>x 7</u>	21 (3x7)	140 20x7	<u>161</u>	<p>Develop understanding of and practice <b>the formal written method of short division with exact answers</b>, supported by practical materials if required</p> <table style="margin-left: 40px;"> <tr><td>14</td></tr> <tr><td>7 <math>\overline{) 98}</math></td></tr> <tr><td>2</td></tr> </table> <p>Answer: 14</p>	14	7 $\overline{) 98}$	2
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2										
<p>Leading to the formal written method to multiply two and three digit numbers by a one digit number.</p> <table style="margin-left: 40px;"> <tr><td>TU</td></tr> <tr><td>23</td></tr> <tr><td><u>x 7</u></td></tr> <tr><td>161</td></tr> <tr><td>2</td></tr> </table> <table style="margin-left: 120px;"> <tr><td>342</td></tr> <tr><td>x 7</td></tr> <tr><td><u>2394</u></td></tr> <tr><td>21</td></tr> </table> <p>Answer: 2394</p>	TU	23	<u>x 7</u>	161	2	342	x 7	<u>2394</u>	21	
TU										
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## Calculation Methods for Year 5

### YEAR 5

Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
<p>Children should calculate mentally with large numbers when nature of numbers makes this appropriate e.g. <math>12,462 - 2,300 = 10,162</math>. They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but may be able to work without jottings. They may use the following strategies.</p>				
Count up to find a small difference			$705 - 287 =$ $8006 - 2993 =$	
Compensating to add/subtract numbers close to a multiple of 10	$346 + 59 =$ $346 + 60 - 1 =$ $406 - 1 = 405$  $478 + 71 =$ $478 + 70 + 1 = 549$	  	$425 - 58 =$ $425 - 60 + 2 =$ $365 + 2 = 367$  $583 - 71 =$ $583 - 70 - 1 =$	  
Partition using multiples of 10. Partition both numbers or just the second.	$324 + 58 =$ $320 + 50 = 370$ $4 + 8 = 12$ or $324 + 50 = 374$ $374 + 8 = 382$		$428 - 43 =$ $428 - 40 - 3 =$	

	Addition	Subtraction
<p>Most children use the formal written methods of columnar addition and subtraction with appropriate <b>whole numbers of more than 4 digits</b></p>		
<p>Most children use the formal written methods.</p> <p>Use the expanded methods to support children who are not ready for the compact written method to understand and move towards use of the formal methods.</p> <p>Place value materials and representations may support children to understand the written procedure if required.</p>	$\begin{array}{r} 587 \\ + 475 \\ \hline 1062 \\ \hline \end{array}$ $\begin{array}{r} 587 \\ +475 \\ \hline 12 \\ 150 \\ \hline 900 \\ \hline 1062 \end{array}$	$\begin{array}{r} 1456 \\ - 753 \\ \hline 47 \text{ (800)} \\ 200 \text{ (1000)} \\ \hline 456 \text{ (1456)} \\ \hline 703 \\ \hline \end{array}$ $\begin{array}{r} 67^{14}5^{14} \\ - 286 \\ \hline 468 \end{array}$

**Continue to practice and apply all tables facts and commit to memory.**

Multiplication		Division	
<b>Signs &amp; Symbols</b> Use signs and symbols to complete questions using known and derived facts eg $9 \times 7 = \ell$ $\ell = 60 \times 2$ $\ell \times 7 = 63$ $120 = \ell \times 2$ $9 \times \ell = 63$ $120 = 60 \times \ell$ $\ell \times \ell = 63$ $120 = \ell \times \ell$		<b>Signs &amp; Symbols</b> Use signs and symbols to complete questions using known and derived facts eg $56 \div 7 = \ell$ $1600 \div 2 = \ell$ $\ell \div 7 = 8$ $\ell \div 2 = 800$ $56 \div \ell = 8$ $1600 \div \ell = 800$ $\ell \div \ell = 8$ $\ell \div \ell = 800$	
<b>Number Sentences</b> Use number sentences to show mental strategy used. eg (i) $36 \times 50 = 1800$ $36 \times 100 = 3600$ $3600 \div 2 = 1800$  (ii) $15 \times 6 = 90$ $15 \times 3 = 45$ $45 \times 2 = 90$		<b>Number Sentences</b> Use number sentences to show mental strategy used. e.g.(i) $198 \div 6 =$ $(180 + 18) \div 6 =$ $30 + 3 = 33$ e.g.(ii) $345 \div 15 =$ $(300 + 45) \div 15 =$ $20 + 3 = 23$	
Most children • <b>Multiply numbers up to 4 digits by a one or two digit number using a formal written method, including long multiplication for two-digit numbers</b>		Most children <b>Divide number up to 4 digits by a one-digit number, using the formal written method of short division and interpret remainders appropriately for the context</b>	
<b>Short multiplication</b>  $342 \times 7$ becomes $\begin{array}{r} 342 \\ \times 7 \\ \hline 2394 \\ \hline 21 \end{array}$ Answer: 2394	<b>Long multiplication for two digit numbers</b>  $24 \times 16$ becomes $\begin{array}{r} 24 \\ \times 16 \\ \hline 144 \\ 240 \\ \hline 384 \end{array}$ Answer: 384	<b>Short division of number up to 4 digits</b>  $432 \div 5$ becomes $496 \div 11$ becomes $\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \\ \underline{40} \phantom{0} \\ 32 \\ \underline{30} \\ 2 \end{array}$ Answer: 86 remainder 2 $\begin{array}{r} 45 \text{ r}1 \\ 11 \overline{) 496} \\ \underline{44} \phantom{0} \\ 56 \\ \underline{55} \\ 1 \end{array}$ Answer: $45 \frac{1}{11}$	
Expanded methods $\begin{array}{r} \text{HTU} \\ 346 \\ \times 9 \\ \hline 54 \text{ (} 6 \times 9 \text{)} \\ 360 \text{ (} 40 \times 9 \text{)} \\ \hline 2700 \text{ (} 300 \times 9 \text{)} \\ \hline 3114 \end{array}$			

## Calculation Methods for Year 6

### YEAR 6

Strategy	Addition sentences	Number lines	Subtraction sentences	Number lines
<p>Children should continue to calculate mentally with large numbers and decimals when nature of numbers makes this appropriate e.g. <math>12,462 - 2,300 = 10,162</math>. They should use number bonds and place value to make efficient steps in their calculations. They may sometimes use a number line to record their thinking but will often be able to work without jottings. They may use the following strategies.</p>				
Count up to find a small difference			$8004 - 2785 =$	
Compensating to add/subtract numbers close to a multiple of 10 and whole numbers when working with decimals	$7.5 + 0.9 =$ $7.5 + 1.0 - 0.1 =$		$19.3 - 2.9 =$ $19.3 - 3.0 + 0.1 =$	
Partition using multiples of 10. Partition both numbers or just the second.	$540 + 280 =$ $540 + 200 + 80 =$		$276 - 153 =$ $276 - 100 - 50 - 3 =$	

	Addition	Subtraction
<p>Most children use the formal written methods of columnar addition and subtraction.</p>		
<p>Most children use the formal written methods. Extend practice to numbers with any number of digits and to two or three decimal places.</p> <p>Use expanded method for those who are not ready for the formal written method and support understanding through the use of place value materials and smaller numbers as appropriate.</p>	$\begin{array}{r} 7648 \\ +1486 \\ \hline 9134 \\ \hline \end{array}$	$\begin{array}{r} 5\ 13\ 1 \\ 6\ 4\ 6\ 7 \\ - 2\ 6\ 8\ 4 \\ \hline 3\ 7\ 8\ 3 \end{array}$

Multiplication		Division	
Continue to practice and complete multiplications and divisions			
$8 \times 9 = \text{pencil}$ $\text{pencil} \times 9 = 72$ $8 \times \text{pencil} = 72$ $\text{pencil} \times \text{pencil} = 72$	$370 \times 2 = \text{pencil}$ $176 \times 2 = \text{pencil}$ $\text{pencil} \times 2 = 3.9$	$72 \div 9 = \text{pencil}$ $72 \div \text{pencil} = 8$ $\text{pencil} \div 9 = 8$ $\text{pencil} \div \text{pencil} = 8$	$1750 \div \text{pencil} = 875$ $570 \div 2 = \text{pencil}$ $\text{pencil} \div 2 = 0.87$
Use number sentences to show mental strategy used. eg $38 \times 25 =$ $38 \times 100 = 3800$ $3800 \div 4 = 950$ eg $35 \times 18 = 630$ $35 \times 6 = 210$ $210 \times 3 = 630$		Use number sentences to show mental strategy used. e.g.(i) $198 \div 6 =$ $(180 + 18) \div 6 =$ $30 + 3 = 33$ e.g.(ii) $345 \div 15 =$ $(300 + 45) \div 15 =$ $20 + 3 = 23$	
Most pupils multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication		Most pupils: <ul style="list-style-type: none"> <li>divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context.</li> <li>divide numbers up to 4 digits by a two-digit whole number using the formal written method of short division where appropriate and interpret remainders according to context.</li> </ul>	
<p><math>2741 \times 6</math> becomes</p> <pre>   2 7 4 1   ×     6   -----   1 6 4 4 6   -----   4 2 </pre> <p>Answer: 16 446</p>	<p><math>124 \times 26</math> becomes</p> <pre>     1 2   1 2 4   ×   2 6   -----   7 4 4   2 4 8 0   -----   3 2 2 4   -----   1 1 </pre> <p>Answer: 3224</p>	<p><math>432 \div 5</math> becomes</p> <pre>       8 6 r2     5   4 3 2       4 0       ---       3 2       3 0       ---         2 </pre> <p>Answer: 86 remainder 2</p>	<p><math>432 \div 15</math> becomes</p> <pre>       2 8 r12     15   4 3 2       3 0 0       ---       1 3 2       1 2 0       ---         1 2 </pre> <p>Answer: 28 remainder 12</p>
		<p><math>432 \div 15</math> becomes</p> <pre>       2 8     15   4 3 2       3 0 0  <math>15 \times 20</math>       ---       1 3 2       1 2 0  <math>15 \times 8</math>       ---         1 2 </pre> <p><math>\frac{12}{15} = \frac{4}{5}</math></p> <p>Answer: <math>28 \frac{4}{5}</math></p>	<p><math>432 \div 15</math> becomes</p> <pre>       2 8 . 8     15   4 3 2 . 0       3 0 0       ---       1 3 2       1 2 0       ---         1 2 0         1 2 0         ---           0 </pre> <p>Answer: 28.8</p>
Standard written methods			
Short multiplication: ThHTU x U			
<pre>       4346     ×   8     -----     6 x 8   48     40 x 8  320     300 x 8 2400     4000 x 8 32000     -----     34768 </pre>			